

$$\begin{aligned}
 &= \int_0^1 \int_0^1 \left(\frac{1}{2} + (y+z) \right) dy dz \\
 &= \int_0^1 \left(\frac{1}{2} y + \frac{y^2}{2} + zy \right) \Big|_0^1 dz \\
 &= \int_0^1 \left(\frac{1}{2} + \frac{1}{2} + z \cdot 1 \right) dz = \int_0^1 (1+z) dz \\
 &= z + \frac{z^2}{2} \Big|_0^1 = 1 + \frac{1}{2} = \frac{3}{2}
 \end{aligned}$$

2. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z) dz dx dy$

$$\rightarrow \int_0^3 \int_0^2 \int_0^1 (x+y+z) dz dx dy$$

$$= \int_0^3 \int_0^2 \left((x+y)z + \frac{z^2}{2} \right) \Big|_0^1 dx dy$$

$$= \int_0^3 \int_0^2 \left[(x+y)(1-0) + \frac{1}{2}(1-0) \right] dx dy$$

$$= \int_0^3 \int_0^2 \left(x+y+\frac{1}{2} \right) dx dy$$

$$= \int_0^3 \left[\frac{x^2}{2} + \left(y + \frac{1}{2} \right) x \right] \Big|_0^2 dy$$

$$= \int_0^3 \left[\frac{1}{2}(4-0) + \left(y + \frac{1}{2} \right) (2-0) \right] dy$$

$$= \int_0^3 (2+2y+1) dy$$

$$= \int_0^3 (2y+3) dy = \frac{2y^2}{2} + 3y \Big|_0^3$$

$$= (9-0) + 3(3-0)$$

$$= 18$$

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3. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz dx dy dz$

$$\rightarrow \int_0^1 \int_0^1 \int_0^y xyz dx dy dz = \int_0^1 \int_0^1 \frac{x^2}{2} \Big|_0^y yz dy dz$$

$$= \int_0^1 \int_0^1 yz \frac{1}{2} (y^2 - 0) dy dz$$

$$= \frac{1}{2} \int_0^1 \int_0^1 zy^3 dy dz = \frac{1}{2} \int_0^1 z \frac{y^4}{4} \Big|_0^1 dz$$

$$= \frac{1}{8} \int_0^1 z(1-0) dz = \frac{1}{8} \frac{z^2}{2} \Big|_0^1$$

$$= \frac{1}{16}$$

4. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$

$$\rightarrow \int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx = \int_0^a \int_0^x \int_0^{x+y} e^x e^y e^z dz dy dx$$

$$= \int_0^a \int_0^x \left[e^x e^y e^z \right]_0^{x+y} dy dx = \int_0^a \int_0^x e^x e^y [e^{x+y} - e^0] dy dx$$

$$= \int_0^a \int_0^x (e^{2x} e^{2y} - e^x e^y) dy dx = \int_0^a e^{2x} \left[\frac{e^{2y}}{2} - e^x e^y \right]_0^x dx$$

$$= \int_0^a \left\{ \frac{e^{2x}}{2} [e^{2x} - e^0] - e^x [e^x - e^0] \right\} dx$$

$$= \int_0^a \left(\frac{1}{2} e^{4x} - \frac{1}{2} e^{2x} - e^{2x} + e^x \right) dx$$

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$$\begin{aligned}
 &= \int_0^a \left(\frac{1}{2} e^{4x} - \frac{3}{2} e^{2x} + e^x \right) dx \\
 &= \left[\frac{1}{2} \frac{e^{4x}}{4} - \frac{3}{2} \frac{e^{2x}}{2} + e^x \right]_0^a \\
 &= \frac{1}{8} (e^{4a} - 1) - \frac{3}{4} (e^{2a} - 1) + (e^a - 1) \\
 &= \frac{1}{8} e^{4a} - \frac{3}{4} e^{2a} + e^a - \frac{3}{8}
 \end{aligned}$$

5. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dz \, dy \, dx$

$$\begin{aligned}
 \int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dz \, dy \, dx &= \int_0^1 \int_0^{1-x} xy \left[\frac{z^2}{2} \right]_0^{1-x-y} dy \, dx \\
 &= \frac{1}{2} \int_0^1 \int_0^{1-x} xy [(1-x-y)^2 - 0] dy \, dx \\
 &= \frac{1}{2} \int_0^1 \int_0^{1-x} xy [(1-x)^2 + y^2 - 2y(1-x)] dy \, dx \\
 &= \frac{1}{2} \int_0^1 \int_0^{1-x} [x(1-x)^2 y + xy^3 - 2x(1-x)y] dy \, dx \\
 &= \frac{1}{2} \int_0^1 \left[x(1-x)^2 \frac{y^2}{2} + x \frac{y^4}{4} - 2x(1-x) \frac{y^3}{3} \right]_0^{1-x} dx \\
 &= \frac{1}{2} \int_0^1 \left[\frac{x(1-x)^4}{2} + \frac{x(1-x)^4}{4} - \frac{2x(1-x)^4}{3} \right] dx \\
 &= \frac{1}{2} \int_0^1 x(1-x)^4 \left(\frac{1}{2} + \frac{1}{4} - \frac{2}{3} \right) dx \\
 &= \frac{1}{2} \int_0^1 x(1-x)^4 \left(\frac{6+3-8}{12} \right) dx = \frac{1}{24} \int_0^1 x(1-x)^4 dx
 \end{aligned}$$

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$$\begin{aligned}
 &= \frac{1}{24} \left[x \frac{(1-x)^5}{5(-1)} \right]_0^1 - \int_0^1 \frac{(1-x)^5}{5(-1)} dx \\
 &= \frac{1}{24} \left[-\frac{1}{5}(0-0) + \frac{1}{5} \frac{(1-x)^6}{6(-1)} \right]_0^1 \\
 &= \frac{1}{24} \left[-\frac{1}{30}(0-1) \right] = \frac{1}{720}
 \end{aligned}$$

6. Evaluate $\int_{-1}^1 \int_{x-z}^{x+z} \int_{-1}^z (x+y+z) dy dx dz$

$$\begin{aligned}
 \rightarrow \int_{-1}^1 \int_{x-z}^{x+z} \int_{-1}^z (x+y+z) dy dx dz &= \int_{-1}^1 \int_{x-z}^{x+z} \left[(x+z)y + \frac{y^2}{2} \right]_{x-z}^{x+z} dx dz \\
 &= \int_{-1}^1 \int_{x-z}^{x+z} \left\{ (x+z)[x+z - (x-z)] + \frac{1}{2} [(x+z)^2 - (x-z)^2] \right\} dx dz \\
 &= \int_{-1}^1 \int_{x-z}^{x+z} \left((x+z)2z + \frac{1}{2}(4xz) \right) dx dz \\
 &= \int_{-1}^1 \int_{x-z}^{x+z} (2xz + 2z^2 + 2xz) dx dz = \int_{-1}^1 \int_{x-z}^{x+z} (2z^2 + 4xz) dx dz \\
 &= \int_{-1}^1 \left[2z^2 x + 4z \frac{x^2}{2} \right]_{x-z}^{x+z} dz = \int_{-1}^1 (2z^3 + 2z^3) dz = \int_{-1}^1 4z^3 dz \\
 &= 4 \left[\frac{z^4}{4} \right]_{-1}^1 = (1^4 - (-1)^4) = 0
 \end{aligned}$$

7. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz dy dx}{\sqrt{a^2-x^2-y^2-z^2}}$

$$\rightarrow \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz dy dx}{\sqrt{a^2-x^2-y^2-z^2}} = \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^k \frac{dz dy dx}{\sqrt{k^2-z^2}}$$

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Where $k^2 = a^2 - x^2 - y^2$

$$\begin{aligned}
 &= \int_0^a \int_0^{\sqrt{a^2-x^2}} \sin^{-1}\left(\frac{z}{k}\right) \Big|_0^k dy dx \\
 &= \int_0^a \int_0^{\sqrt{a^2-x^2}} \left[\sin^{-1}\left(\frac{k}{k}\right) - \sin^{-1}(0) \right] dy dx \\
 &= \int_0^a \int_0^{\sqrt{a^2-x^2}} \left[\frac{\pi}{2} - 0 \right] dy dx = \frac{\pi}{2} \int_0^a y \Big|_0^{\sqrt{a^2-x^2}} dx \\
 &= \frac{\pi}{2} \int_0^a \left[\sqrt{a^2-x^2} \right] dx \\
 &= \frac{\pi}{2} \left[\frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2} \sqrt{a^2-x^2} \right]_0^a \\
 &= \frac{\pi}{2} \left[\frac{a^2}{2} \left(\frac{\pi}{2} - 0 \right) + (0-0) \right] = \frac{\pi^2 a^2}{8} \quad \blacksquare
 \end{aligned}$$

8. Evaluate $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sec\theta} r^3 \sin\theta dr d\theta d\phi$

$$\begin{aligned}
 \Rightarrow \text{Let } I &= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sec\theta} r^3 \sin\theta dr d\theta d\phi \\
 &= \int_0^{\pi/2} \int_0^{\pi/4} \left[\frac{r^4}{4} \right]_0^{\sec\theta} \sin\theta d\theta d\phi \\
 &= \frac{1}{4} \int_0^{\pi/2} \int_0^{\pi/4} (\sec^4\theta - 0) \sin\theta d\theta d\phi \\
 &= \frac{1}{4} \int_0^{\pi/2} \int_0^{\pi/4} \sec^4\theta \sin\theta d\theta d\phi = \frac{1}{4} \int_0^{\pi/2} \int_0^{\pi/4} \frac{\sin\theta}{\cos^4\theta} d\theta d\phi
 \end{aligned}$$

Put $\cos\theta = t \Rightarrow \sin\theta d\theta = -dt$

If $\theta = 0 \Rightarrow t = 1$, if $\theta = \frac{\pi}{4} \Rightarrow t = \frac{1}{\sqrt{2}}$

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$$\begin{aligned}
 I &= \frac{1}{4} \int_0^{\pi/2} \int_1^{1/\sqrt{2}} \frac{1}{t^4} (-dt) d\phi \\
 &= \frac{1}{4} \int_0^{\pi/2} \int_1^{1/\sqrt{2}} t^{-4} dt d\phi = -\frac{1}{4} \int_0^{\pi/2} \left. \frac{t^{-3}}{-3} \right|_1^{1/\sqrt{2}} d\phi \\
 &= \frac{1}{12} \int_0^{\pi/2} \left[\left(\frac{1}{\sqrt{2}} \right)^{-3} - 1 \right] d\phi = \frac{1}{12} (2\sqrt{2} - 1) \phi \Big|_0^{\pi/2} \\
 &= \frac{1}{12} (2\sqrt{2} - 1) \left(\frac{\pi}{2} - 0 \right) \\
 &= \frac{\pi}{24} (2\sqrt{2} - 1) \quad \blacksquare
 \end{aligned}$$

9. If R is the region bounded by the co-ordinate planes and the planes $x=a$, $y=a$, $z=a$, in the first quadrant then show that

$$\iiint_R (x^2 + y^2 + z^2) dx dy dz = a^5$$

■ Let $I = \iiint_R (x^2 + y^2 + z^2) dx dy dz$

Here $x: 0 \rightarrow a$

$y: 0 \rightarrow a$

$z: 0 \rightarrow a$

$$\begin{aligned}
 \therefore I &= \int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2) dx dy dz \\
 &= \int_0^a \int_0^a \left(\frac{x^3}{3} + (y^2 + z^2)x \right) \Big|_0^a dy dz \\
 &= \int_0^a \int_0^a \left[\frac{a^3}{3} + (y^2 + z^2)a \right] dy dz \\
 &= \int_0^a \left(\frac{a^3}{3} y + a \frac{y^3}{3} + az^2 y \right) \Big|_0^a dz
 \end{aligned}$$

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$$\begin{aligned}
 &= \int_0^a \left(\frac{a^4}{3} + \frac{a^4}{3} + a^2 z^2 \right) dz = \int_0^a \left(\frac{2a^4}{3} + a^2 z^2 \right) dz \\
 &= \frac{2a^4}{3} z + a^2 \frac{z^3}{3} \Big|_0^a = \frac{2a^5}{3} + \frac{a^5}{3} = \frac{3a^5}{3} = a^5
 \end{aligned}$$

Exercises

I. Evaluate the following triple integrals.

1)
$$\int_0^2 \int_1^3 \int_1^2 xy^2 z dz dy dx$$

2)
$$\int_{-3}^3 \int_0^1 \int_1^2 (x+y+z) dx dy dz$$

3)
$$\int_0^1 \int_{y^2}^{1-x} \int_0^x x dz dy dx$$

4)
$$\int_0^1 \int_0^{1-x} \int_0^y e^{x+y+z} dz dy dx$$

5)
$$\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{z}} \int_0^{\sqrt{4-z^2}} dy dx dz$$

6)
$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dz dy dx$$

7)
$$\int_0^1 \int_0^1 \int_{\sqrt{x^2+y^2}}^2 xyz dy dx dz$$

8)
$$\int_0^1 \int_1^e \int_0^x \log z dz dx dy$$

9)
$$\int_0^{\frac{\pi}{2}} \int_0^{a \sin \theta} \int_0^a r dr d\theta dz$$

10)
$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$$

11)
$$\int_0^a \int_0^{a-x} \int_0^{a-x-y} (x^2 + y^2 + z^2) dz dy dx$$

12)
$$\int_0^a \int_0^{\sqrt{a^2-z^2}} \int_0^{\sqrt{a^2-y^2-z^2}} (x^2 + y^2 + z^2) dx dy dz$$

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- 13) $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dx dy dz}{(1+x+y+z)^3}$
- 14) $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz dy dx}{\sqrt{a^2-x^2-y^2-z^2}}$
- 15) $\int_0^\infty \int_0^\infty \int_0^\infty e^{-(x+y+z)} dx dy dz$
- 16) $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz dy dx}{(1+x+y+z)^3}$
- 17) $\int_0^{\frac{\pi}{2}} \int_0^a \int_0^{\sqrt{a^2-R^2}} R dz dR d\theta$
- 18) $\int_{r=0}^a \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{\frac{\pi}{2}} r^2 \sin \theta d\phi d\theta dr$

II. If R is the region bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$, show that

(i) $\iiint_R z dx dy dz = \frac{1}{24}$ (ii) $\iiint_R (x+y+z)^9 dx dy dz = \frac{1}{24}$

III. If R is the region $-a \leq x \leq a, -b \leq y \leq b, -c \leq z \leq c$, show that

$\iiint_R (x^2 + y^2 + z^2) dx dy dz = \frac{8}{3} abc(a^2 + b^2 + c^2)$

Answers

- I. 1) 26 2) 12
- 3) $\frac{4}{35}$ 4) $\frac{1}{8}e^4 - \frac{3}{4}e^2 + e - \frac{3}{8}$
- 5) 8π 6) $\frac{1}{48}$
- 7) $\frac{2}{8}$ 8) $\frac{1}{2}(e^2 - 8e + 13)$
- 9) πa^3 10) $\frac{\pi^2}{8}$

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11) $\frac{a^5}{20}$

12) $\frac{a^5}{10}$

13) $\frac{1}{2} \log\left(2 - \frac{5}{8}\right)$

14) $\left(\frac{\pi^2}{8}\right)a^2$

15) 1

16) $\frac{\pi a^3 (2 - \sqrt{2})}{3}$

17) $\frac{5\pi}{64}a^3$

18) $\frac{\pi}{6}a^3$

GAMMA AND BETA FUNCTIONS

In this section we consider two improper integrals called the Gamma function and the Beta function. These integrals are together known as Eulerian integrals and these functions play important role in applied mathematics.

The Gamma Function

The Gamma function is denoted by $\Gamma(n)$ where n is positive real number or negative fraction and is defined by

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

Other Form of Gamma Function

We know that $\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$

Put $x = t^2 \Rightarrow dx = 2t dt$,

If $x = 0 \Rightarrow t = 0$, if $x = \infty \Rightarrow t = \infty$,

Therefore, $\Gamma(n) = 2 \int_0^{\infty} t^{2n-1} e^{-t^2} dt$

This is the other form of Gamma function.

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Recurrence formula

Prove that $\Gamma(n+1) = n\Gamma(n)$

Proof We know that $\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$

Put $n = n+1$

$$\begin{aligned}\Gamma(n+1) &= \int_0^{\infty} x^n e^{-x} dx \\ &= x^n \frac{e^{-x}}{-1} \Big|_0^{\infty} - \int_0^{\infty} nx^{n-1} \frac{e^{-x}}{-1} dx \quad [\text{Using integration by parts}] \\ &= (0-0) + n \int_0^{\infty} x^{n-1} e^{-x} dx \quad \because e^{-\infty} = 0 \\ &= n \int_0^{\infty} x^{n-1} e^{-x} dx \\ &= n\Gamma(n)\end{aligned}$$

This is the recurrence formula for $\Gamma(n)$

Prove that $\Gamma(n) = (n-1)!$, where n is positive integer

Proof We know that $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$

$$\text{Put } n = 1 \quad \Gamma(1) = \int_0^{\infty} e^{-x} x^0 dx = \int_0^{\infty} e^{-x} dx = \left[\frac{e^{-x}}{-1} \right]_0^{\infty} = -1(0-1) = 1$$

$$\Gamma(2) = \Gamma(1+1) = 1\Gamma(1) = 1.1 = 1 = 1!$$

$$\Gamma(3) = \Gamma(2+1) = 2\Gamma(2) = 2.1 = 2!$$

Similarly $\Gamma(4) = 3!$

In general $\Gamma(n) = (n-1)!$

Generalized Recurrence Formula

Prove that $\Gamma(n+k+1) = n(n+1)(n+2)\dots(n+k)\Gamma(n)$

Where n is positive real number or a negative non-integer and k is positive integer.

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Proof We shall prove the relation by mathematical induction.

(1) If $k = 1$ the given result gives,

$$\begin{aligned}\Gamma(n+2) &= n(n+1)\Gamma(n) \\ \text{LHS} &= \Gamma(n+2) = \Gamma(n+1+1) \\ &= (n+1)\Gamma(n+1) = (n+1)n\Gamma(n) \\ &= n(n+1)\Gamma(n) \\ &= \text{RHS}\end{aligned}$$

Therefore, result is true for $k = 1$

(2) Assume the result is true for $k = m$

$$\text{i.e., } \Gamma(n+m+1) = n(n+1)(n+2)\dots(n+m)\Gamma(n) \quad \text{----(1)}$$

(3) If $k = m+1$ the given result gives

$$\begin{aligned}\Gamma(n+m+2) &= n(n+1)(n+2)\dots(n+m)(n+m+1)\Gamma(n) \\ \text{LHS} &= \Gamma(n+m+2) \\ &= \Gamma(n+m+1+1) \\ &= (n+m+1)\Gamma(n+m+1) \\ &= (n+m+1)[n(n+1)(n+2)\dots(n+m)\Gamma(n)] \quad [\text{From equation (1)}] \\ &= n(n+1)(n+2)\dots(n+m)(n+m+1)\Gamma(n) \\ &= \text{RHS}\end{aligned}$$

Therefore, the result is true for $k = m+1$

Hence, the result is true for all values of k

Worked Examples

1. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

► We know that $\Gamma(n) = 2 \int_0^{\infty} x^{2n-1} e^{-x^2} dx$

$$\text{Put } n = \frac{1}{2}, \quad \Gamma\left(\frac{1}{2}\right) = 2 \int_0^{\infty} e^{-x^2} dx = 2 \int_0^{\infty} e^{-y^2} dy$$

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$$\therefore \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) = 2 \int_0^{\infty} e^{-x^2} dx \times 2 \int_0^{\infty} e^{-y^2} dy$$

$$\left[\Gamma\left(\frac{1}{2}\right)\right]^2 = 4 \int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$$

Now, we use the polar coordinates to evaluate the above integral
i.e., $x = r \cos \theta$, $y = r \sin \theta$, $dx dy = r dr d\theta$

given $x: 0 \rightarrow \infty$ $y: 0 \rightarrow \infty$

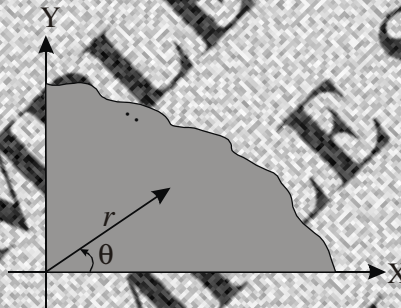


Figure 3.10

From the figure, we have

$$r: 0 \rightarrow \infty, \theta: 0 \rightarrow \pi/2$$

$$\therefore \left[\Gamma\left(\frac{1}{2}\right)\right]^2 = 4 \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} e^{-r^2} r dr d\theta$$

$$\text{Put } r^2 = t \Rightarrow 2r dr = dt \Rightarrow r dr = dt/2$$

$$\text{If } r = 0 \Rightarrow t = 0, \text{ if } r = \infty \Rightarrow t = \infty$$

$$\left[\Gamma\left(\frac{1}{2}\right)\right]^2 = 4 \int_{\theta=0}^{\pi/2} \int_{t=0}^{\infty} e^{-t} \frac{dt}{2} \cdot d\theta$$

$$= 2 \int_0^{\pi/2} \left[\frac{e^{-t}}{-1} \right]_0^{\infty} d\theta = 2 \int_0^{\pi/2} -(0-1) d\theta$$

$$= 2 \int_0^{\pi/2} d\theta = 2\theta \Big|_0^{\pi/2} = 2 \left(\frac{\pi}{2} - 0 \right)$$

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