

Here $m = 3$ $n = 7$ therefore, from case (1), we get

$$I_{3,7} = \frac{6.4.2}{10.8.6} \cdot \frac{1}{4} = \frac{1}{40}$$

2. Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^3 x dx$

$$\Rightarrow I_{5,3} = \int_0^{\frac{\pi}{2}} \sin^5 x \cos^3 x dx = \frac{2}{8} \cdot \frac{1}{6} = \frac{1}{24}$$

3. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^8 x dx$

$$\Rightarrow I_{6,8} = \frac{7.5.3.1}{14.12.10.8} \cdot \frac{5.3.1}{6.4.2} \left(\frac{\pi}{2} \right) = \frac{5\pi}{2^{12}}$$

4. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x dx$

$$\Rightarrow I_{6,5} = \frac{4.2}{11.9} \cdot \frac{5.3.1}{7.5.3.1} = \frac{8}{693}$$

5. Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^5 x dx$

$$\Rightarrow I_{5,5} = \frac{4.2}{10.8} \cdot \frac{1}{6} = \frac{1}{60}$$

6. Evaluate $\int_0^{\frac{\pi}{4}} \sin^4 4\theta \cos^2 2\theta d\theta$

$$\begin{aligned} \Rightarrow I &= \int_0^{\frac{\pi}{4}} \sin^4 4\theta \cos^2 2\theta d\theta = \int_0^{\frac{\pi}{4}} (2\sin 2\theta \cos 2\theta)^4 \cos^2 2\theta d\theta \\ &= 2^4 \int_0^{\frac{\pi}{4}} \sin^4 2\theta \cos^6 2\theta d\theta \end{aligned}$$

Put $2\theta = x \Rightarrow d\theta = dx/2$

If $\theta = 0 \Rightarrow x = 0$, If $\theta = \pi/4 \Rightarrow x = \pi/2$

$$\therefore I = 2^4 \int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x \frac{dx}{2} = 2^3 \frac{5.3.1}{10.8.6} \cdot \frac{3.1}{4.2} \frac{\pi}{2} = \frac{3\pi}{64}$$

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7. Evaluate $\int_0^{\pi} x \sin^4 x \cos^6 x \, dx$

► Let $I = \int_0^{\pi} x \sin^4 x \cos^6 x \, dx$

$$= \int_0^{\pi} (\pi - x) \sin^4 (\pi - x) \cos^6 (\pi - x) \, dx$$

$$\left[\because \int_0^a f(x) \, dx = \int_0^a (a-x) \, dx \right]$$

$$I = \int_0^{\pi} (\pi - x) \sin^4 x \cos^6 x \, dx$$

$$= \pi \int_0^{\pi} \sin^4 x \cos^6 x \, dx - \int_0^{\pi} x \sin^4 x \cos^6 x \, dx$$

$$I = \pi \int_0^{\pi} \sin^4 x \cos^6 x \, dx - I$$

$$2I = \pi \int_0^{\pi} \sin^4 x \cos^6 x \, dx$$

$$2I = 2\pi \int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x \, dx$$

$$I = \pi \left[\frac{(5.3.1)(3.1)}{10.8.6.4.2} \cdot \frac{\pi}{2} \right] = \frac{3\pi^2}{512}$$

8. Evaluate $\int_0^1 x^6 \sqrt{1-x^2} \, dx$

► Let $I = \int_0^1 x^6 \sqrt{1-x^2} \, dx$

$$\text{Put } x = \sin \theta \Rightarrow dx = \cos \theta \, d\theta$$

$$\text{If } x=0 \Rightarrow \theta=0, \text{ If } x=1 \Rightarrow \theta=\pi/2$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \sin^6 \theta \sqrt{1-\sin^2 \theta} \cos \theta \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^6 \theta \cos^2 \theta \, d\theta$$

$$= \frac{5.3.1}{8.6.4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{5\pi}{256}$$

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9. Evaluate $\int_0^1 x^2(1-x^2)^{\frac{3}{2}} dx$

$$\Rightarrow \text{Let } I = \int_0^1 x^2(1-x^2)^{\frac{3}{2}} dx$$

Put $x = \sin \theta$

If $x = 0 \Rightarrow \theta = 0$, If $x = 1 \Rightarrow \theta = \pi/2$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 \theta (1 - \sin^2 \theta)^{\frac{3}{2}} \cos \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^4 \theta d\theta$$

$$= \frac{(3.1)(1)}{6.4.2} \cdot \frac{\pi}{2} = \frac{\pi}{32}$$

10. Evaluate $\int_0^1 x^3(1-x^2)^{5/2} dx$

$$\Rightarrow \text{Let } I = \int_0^1 x^3(1-x^2)^{5/2} dx$$

Put $x = \sin \theta \Rightarrow dx = \cos \theta d\theta$

If $x = 0 \Rightarrow \theta = 0$, If $x = 1 \Rightarrow \theta = \pi/2$

$$I = \int_0^{\frac{\pi}{2}} \sin^3 \theta (1 - \sin^2 \theta)^{5/2} \cos \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^5 \theta \cdot \cos^6 \theta d\theta$$

$$= \frac{(5.3.1)(4.2)}{11.9.7.5.3.1} = \frac{8}{693}$$

11. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$

$$\Rightarrow \text{Let } I = \int_0^{2a} x^2 \sqrt{2ax - x^2} dx$$

Put $x = 2a \sin^2 \theta \Rightarrow dx = 4a \sin \theta \cdot \cos \theta d\theta$

If $x = 0 \Rightarrow \theta = 0$, If $x = 2a \Rightarrow \theta = \pi/2$

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$$\begin{aligned}
 I &= \int_0^{\frac{\pi}{2}} (2a \sin^2 \theta)^2 \sqrt{2a(2a \sin^2 \theta) - (2a \sin^2 \theta)^2} 4a \cos \theta \sin \theta \cdot d\theta \\
 &= \int_0^{\frac{\pi}{2}} 4a^2 \sin^4 \theta \cdot 2a \sin \theta \sqrt{1 - \sin^2 \theta} \cdot 4a \cos \theta \cdot \sin \theta \cdot d\theta \\
 &= 32a^4 \int_0^{\frac{\pi}{2}} \sin^6 \theta \cdot \cos^2 \theta d\theta = 32a^4 \frac{(1)(5)(3)(1)}{8 \cdot 6 \cdot 4 \cdot 2} \cdot \frac{\pi}{2} = \frac{5a^4 \pi}{8}
 \end{aligned}$$

12. Evaluate $\int_0^{\infty} \frac{x^6}{(1+x^2)^{9/2}} dx$

► Let $I = \int_0^{\infty} \frac{x^6}{(1+x^2)^{9/2}} dx$

Put $\therefore x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

If $x = 0 \Rightarrow \theta = 0$, If $x = \infty \Rightarrow \theta = \pi/2$

$$\begin{aligned}
 \therefore I &= \int_0^{\frac{\pi}{2}} \frac{\tan^6 \theta}{(\sec^2 \theta)^{9/2}} \sec^2 \theta d\theta = \int_0^{\frac{\pi}{2}} \tan^6 \theta \cdot \cos^7 \theta d\theta \\
 &= \int_0^{\frac{\pi}{2}} \sin^6 \theta \cdot \cos \theta d\theta = \frac{1}{7}
 \end{aligned}$$

Exercises

Evaluate the following integrals

1) $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^8 x dx$

2) $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx$

3) $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^2 x dx$

4) $\int_0^{\frac{\pi}{2}} x \sin x \cos^4 x dx$

5) $\int_0^{\pi} \sin^4(\theta/2) \cos^3(\theta/2) d\theta$

6) $\int_0^{\frac{\pi}{2}} \sin^2 x (\sin^3 x + \cos^3 x) dx$

7) $\int_0^{\frac{\pi}{6}} \sin^2 6\theta \cos^4 3\theta d\theta$

8) $\int_0^1 x^{\frac{3}{2}} (1-x)^{\frac{3}{2}} dx$

9) $\int_0^{\pi} x \sin^3 x \cos^4 x dx$

10) $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^5 x dx$

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11) $\int_0^\pi x \sin^6 x \cos^4 x dx$

12) $\int_0^1 x^{\frac{3}{2}} \sqrt{1-x} dx$

13) $\int_0^{2a} x^3 \sqrt{2ax-x^2} dx$

14) $\int_0^{\frac{1}{2}} x^3 \sqrt{1-4x^2} dx$

15) $\int_0^\infty \frac{x^5}{(1+x^2)^6} dx$

16) $\int_0^\infty \frac{x^3}{(x^2+4)^{\frac{3}{2}}} dx$

17) $\int_0^\infty \frac{x^2}{(x^2+a^2)^5} dx$

18) $\int_0^4 x^{\frac{9}{2}} (4-x)^{\frac{1}{2}} dx$

19) $\int_0^2 x^3 \sqrt{2-x} dx$

20) $\int_0^\infty \frac{x^4}{(1+x^2)^4} dx$

21) If n is a positive integer, then show that

$$\int_0^{2a} x^n \sqrt{2ax-x^2} dx = \frac{(2n+1)!}{(n+2)! n!} \frac{a^{n+2}}{2^n} \pi$$

Answers

- | | | |
|-------------------------------|--------------------------|----------------------|
| 1) $\frac{5\pi}{4096}$ | 2) $\frac{2}{35}$ | 3) $\frac{8}{105}$ |
| 4) $\frac{\pi}{5}$ | 5) $\frac{4}{35}$ | 6) $\frac{2}{3}$ |
| 7) $\frac{5\pi}{192}$ | 8) $\frac{3\pi}{128}$ | 9) 0 |
| 10) $\frac{1}{120}$ | 11) $\frac{3\pi^2}{512}$ | 12) $\frac{\pi}{16}$ |
| 13) $\frac{5\pi a^4}{8}$ | 14) $\frac{1}{120}$ | 15) $\frac{1}{7}$ |
| 16) $\frac{1}{2}$ | 17) $\frac{1}{12a^6}$ | 18) 252π |
| 19) $\frac{512\sqrt{2}}{315}$ | 20) $\frac{\pi}{32}$ | |

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